Algebra

- al-jabr is Arabic for "reunion of broken parts"
- from the 9th century to the 21st
- Pioneered by al-Khwārizmī

Multiplying and Dividing Expressions

Recap

- $3 \times a$ is written as 3a, $a \times b$ is written as ab
- Multiplication can happen in any order so we have:
  - $3 \times a$ is the same as $a \times 3$
  - $3 \times (a \times b)$ is the same as $(3 \times a) \times b$: we write these as $3ab$
- $a \times a$ is written as a^2
- We write numbers first and then letters in alphabetical order in a term, e.g. $b \times 2$ × a is written as $2ab$

Multiplying

- What would it be like if we multiplied $4a \times 2ab$?
- Can we do this? We wouldn't be able to add them
- With multiplication/division, our terms don't have to be like terms
- When multiplying 2 terms, we multiply the coefficients then the pronumerals
- So we would have $4a \times 2ab = 8a^2b$

Dividing

- $k \div 3$ is written as $\frac{k}{3}$
- $3 \div k$ is written as $\frac{3}{k}$
- We can cancel common factors in the numerator and denominator
  - e.g. $\frac{8x}{12xy}$
  - e.g. $\frac{{}^{2}\cancel{4}a\,{}^{1}\cancel{b}}{{}^{1}\cancel{2}\,{}^{1}\cancel{b}c} = \frac{2a}{c}$

Multiplying and Dividing Expressions

$k \times 3$	$3k$	For multiplication, we leave off the × symbol
$k \div 3$	$\frac{k}{3}$	Division can be represented as a fraction
$k \times k$	k^2	powers for pronumerals are represented the same as for numbers
$1k$	k	Think how $1 \times 4 = 4$
$c \times a \times 3$	$3ac$	Write the number first, then variables in alphabetical order

- And remember to simplify fractions!
- Now here's challenge question $(3a)^2 - 4a^2 + a \div a - 5a \times a - 1$

Equivalent Algebraic expressions

- So now we know how to add, subtract, multiply and divide expressions
- Let's put it all together to make as many equivalent expressions as we can for:

$$-7x - 4$$

- Can we swap x with another pronumeral?
- In fact, pronumerals replace a few symbols you might have seen in primary
  - ?, ___, □, ◆, or other shapes

Expanding Algebraic Expressions

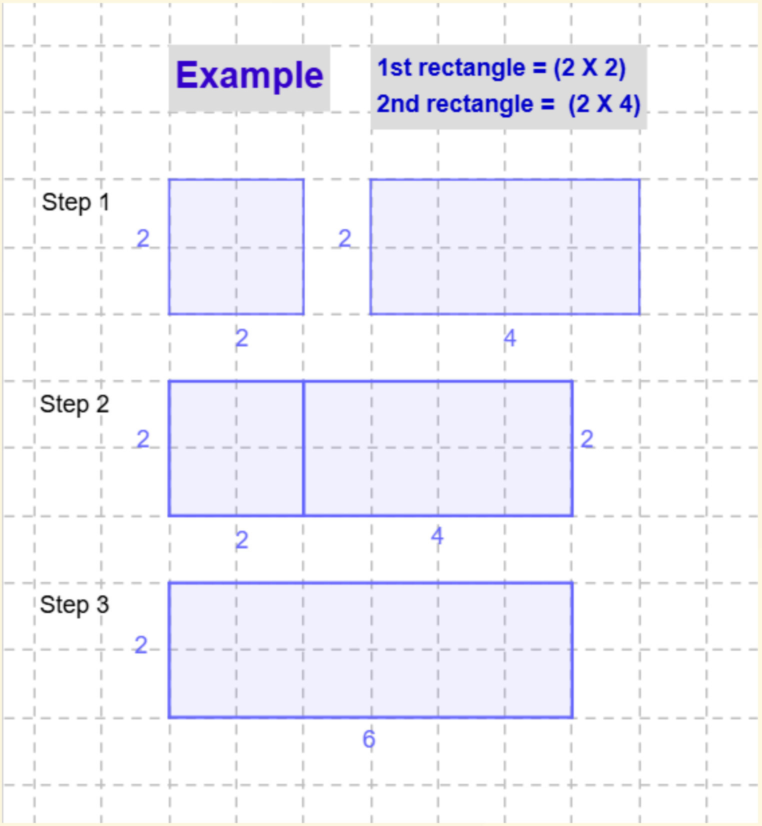
Sometimes we get an expression that looks like:

$$2(x + 4)$$

- What do we do?
- We multiply $2 \times x$ and 2×4
- And we get $2x + 8$
- Why does this work?

The Distributive Principle

a x (b+c) = ab + ac

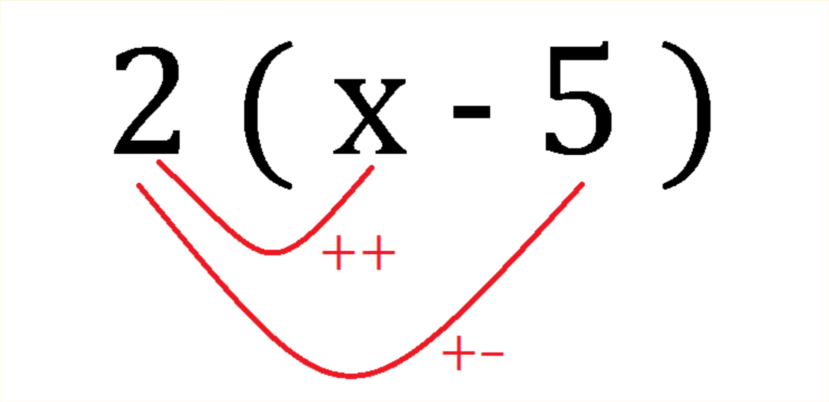


Common Mistakes

- $2(x+5) \neq 2x+5$
  - Sometimes people only multiply the first term, but the brackets mean we multiply both terms
- $-2(x+5) \neq -2x+10$
  - and other positive/negative mixups
- $2(x+5) \neq 2x+7$
  - We have to multiply instead of adding: the 5 is added to the x, not 2
- $2(x+5)+10 \neq 2x+10+20$
  - We only multiply the 2 with what is in the brackets

How to avoid

- I suggest drawing lines between the "outsider" (2) and only the "insiders" (x and 5)
- Also, pay attention to the signs, say them out if it helps
  - e.g., for $-2(x-5)$, "negative times positive", "negative times negative"



Expanding Brackets Bingo

Choose 9 of these answers to put in your grid

$a^2 + 9a$	$9a + 36$	$6a + 6b$	$-15a + 40b$
$-5a - 5b$	$20a + 10$	$64a + 72$	$10a - 45b$
$a^2 + 2a$	$-9a + 9b$	$-6a + 6b$	$-a^2 + 6a$
$5a - 5b$	$5a + 5b$	$-4a + 4b$	$9a - 27$

Next Question

Finish

Linear Equations

- Equations have an equal sign and two sides
- We solve an equation by getting a pronumeral by itself on one side

$$2h + 3 = 11$$

- First we subtract 3 from both sides and get

$$2h = 8$$

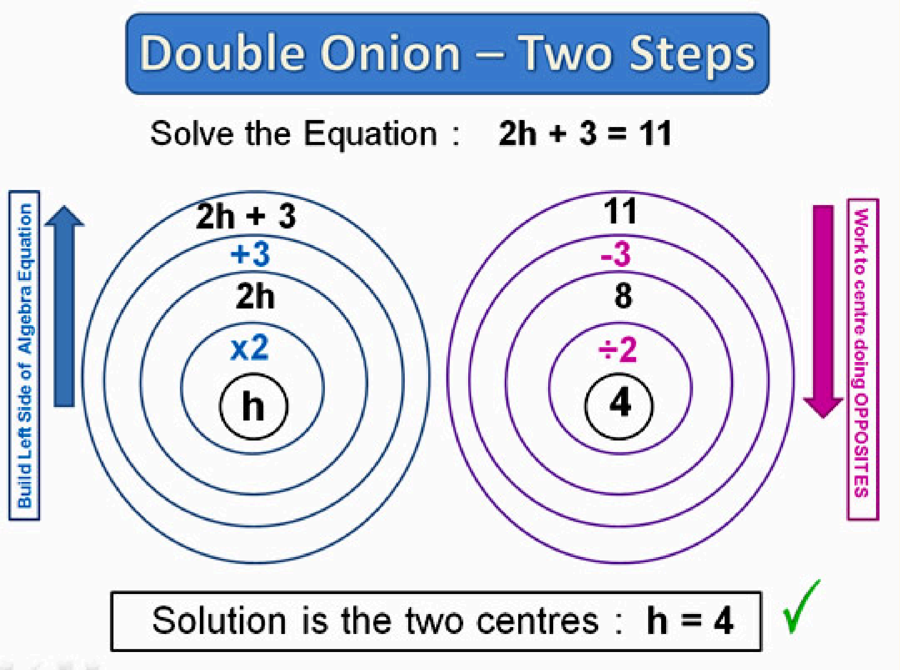
- Then we divide both sides by 2 to get

$$x = 4$$

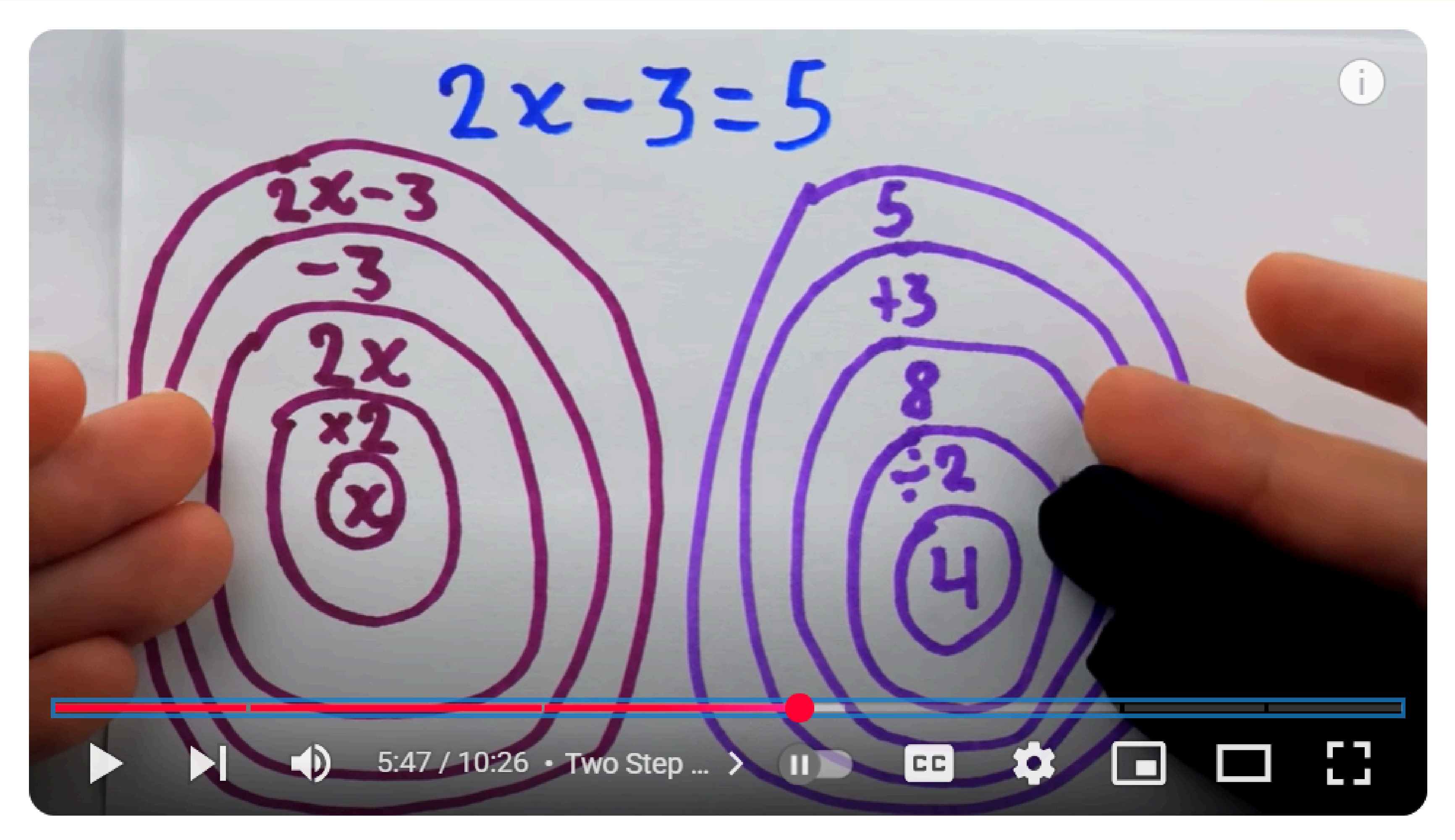
- Remember, what we do to one side, we have to do to both sides https://www.geogebra.org/m/fjekqnuh?lang=en-AU

Reversing Operations

- Why did we start with the constant term?
- Think of the expressions like an **onion**
- x is at the center
- Constant terms are the most outside layer
- Your Turn: Draw the onion for $-3x - 5$

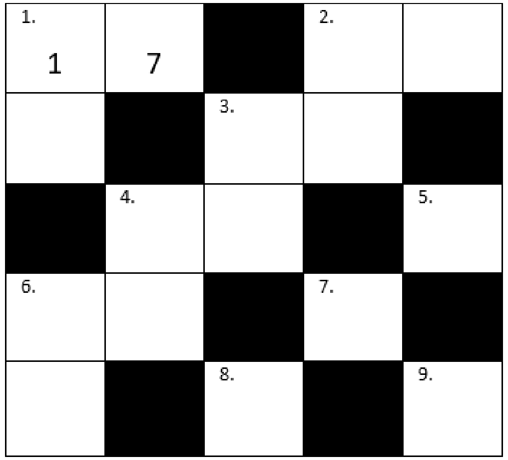


Onion Method



Crossnumber

1. First fill out the grid with numbers
2. Then write an equation for each clue so that the solution is the corresponding number in the grid
3. Once you've written the whole puzzle, copy the clues onto the second copy and hand it to a friend
4. Solve the puzzle from your friend



Across

1. $3x - 1 = 50$
2.
3.
4.

Down

1.
2.
3.
4.

Linear Equations with Fractions

- For $5x = 5$ we divided both sides by 5
- What do we do for $\frac{1}{5}x = 1$?
- That's right, we multiply

$$\frac{1}{5}x \times 5 = 1 \times 5$$

$$\frac{1}{\cancel{5}}x \times \cancel{5} = 5$$

$$x = 5$$

- Explanation: We multiplied because we need to do the opposite of division

Let's try

- $\frac{x}{2} = 8$

- $\frac{x}{2} + 5 = 8$

- $\frac{x+5}{2} = 8$

Linear equations with brackets

- Take this expression: $3(x+4)$
  - How do we expand it?
- We learnt earlier how to expand algebraic expressions using the distributive law
- We multiply $3 \times x$ and 3×4
- And get $3x + 12$
- But do we have to expand the brackets?

To Expand or Not To Expand?

- It depends on the equation
- Let's take a look at $3(x+2)=12$ vs $3(x+4)=14$

Expanding

$$3(x+4)=14$$
$$3x+12=14$$
$$3x=2$$
$$x=\frac{2}{3}$$

Not Expanding

$$3(x+2)=12$$
$$x+2=4$$
$$x=2$$

$3(x+4)=14$ **vs** $3(x+2)=12$

- Why did we pick to expand or not expand?
  - 12 is divisible by 3, 14 is not
- But this does require you know divisibility
- I suggest you go ahead and expand
- Expanding will always get you the right answer
- And it's more straight-forward

Pronumerals on both sides

- Say we have an equation like:
  - $$2x - 6 = 5x - 15$$
- The rule here is that we put all the terms with x on one side and all the constant terms on the other
- Why?
- We can't simplify $5x - 15$ or $2x - 6$
  - (or any pronumeral + constant term)
  - So we gather **like terms** which we *can* simplify

How to gather like terms

❤️ This is wrong actually. Just do the worked example in the booklet

Let's take our example.

$$2x - 6 = 5x - \cancel{15}$$

+15 +15

We start with the constant term -15

$$\cancel{2x} + 9 = 5x$$

-2x -2x

Then we subtract the x with the smaller coefficient

$$9 = 3x$$

÷3 ÷3

Now we get rid of the coefficient

Solving Problems with Equations

1. We uses algebra to "model" a real-world situation.
    - We take the information in the given problem (Circle, Underline, Box)
    - We represent quantities using pronumerals
    - We write an equation representing the situation (Organise)
2. We solve the equation and find the pronumeral's value
3. We add back in any units (e.g. dollars, meters, years, etc.)

Constructing expressions from problem statements

We see from the problem what values can change (these become the **pronumerals**) and which stay the same (these become the **constants**)

Example: A plumber charges a $60 call-out fee plus $50 per hour.

- The call-out fee: constant
- The number of hours: variable

What's the cost of an 8 hour visit? An x hour visit?

- cost = 60 + 50x

Learning Intention

Today we're learning about Formulas

3J	**21 I can substitute values into formulas and find the unknown.** e.g. Substitute the given values into the formula to find the value of the unknown. Round to one decimal place where necessary. **a** $A = \frac{1}{2}xy$ when $x = 7$ and $y = 10$ **b** $V = \pi r^2 h$ when $V = 120$ and $r = 3$	☐
3J	**22 (Extension) I can transpose a formula.** e.g. Transpose the formula $v^2 = u^2 + 2as$ to make u the subject $(u > 0)$.	☐

20.5.25 Revisit at end of lesson

Using Formulas

What are some examples you know of formulas?

- $$\boxed{A = \pi r^2}$$
  - the area, A, of a circle which has radius r
- $$\boxed{V = \pi r^2 h}$$
  - the volume, V, of a cylinder which has radius r, and height h
- $$\boxed{F = \frac{9}{5}C + 32}$$
  - converting degrees Celsius, C, to degrees Fahrenheit, F

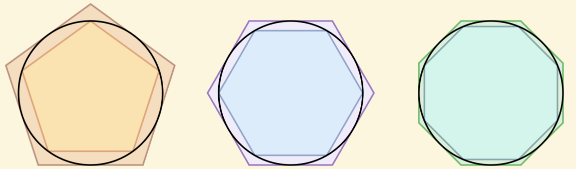
Subjects of Formulas

- The **subject** of a formula is a pronumeral that usually sits on its own on the LHS. (It's like a recipe)
- We can solve a pronumeral in a formula by substituting numbers for all other pronumerals e.g. $V = \pi \times (5\mathrm{cm})^2 \times 10\mathrm{cm}$
  - We may have to solve a simple equation if we're trying to find something other than the subject
- But! We can rearrange a formula to change the subject.
  - We do it same as when solving an equation - add, subtract, multiply, divide, e.g. $h = \frac{V}{\pi r^2}$

Extra: Where Do Formulas Come From?

And who thought they were a good idea??

- Ancient Egyptian mathematicians! The first formulas were from ~1800 **BC** and calculated areas of rectangles, triangles, etc.
- Greeks Archimedes and Eudoxus found formulas for circles and curves through the method of exhaustion



- Formulas have been found by mathematicians for more than centuries: Pythagoras, Heron, Euler....

Revision

Solving equations

If we have fractions (page 29):

- multiply both sides by the denominator
- eg. $\frac{x+3}{5} = 7$
  - Remember: The 7 will be multiplied by 5, the $\frac{x+3}{5}$ will become $x + 3$

If we have brackets (page 34):

- expand by multiplying insiders by the outsider
- eg. $3(x+4)+5=19$
  - Remember: draw lines, check the signs and multiply only and every term in the brackets

If we have pronumerals on both sides (page 38)

- Subtract the x's from the side with bigger coefficient
- eg. $5x = 2x + 3$
- Then we get a normal equation

If there's a combination of these three:

- Deal with what's furthest away from x first

Checking our answers

To check, we can substitute the solution back into the equation

- $\frac{x+3}{5} = 7$
- $3(x+4)+5=19$
- $5x = 2x + 3$

- Substitute $x = 32$ back in
- Substitute $x = \frac{2}{3}$ back in
- Substitute $x = 1$ back in

- If the left and right sides of the equation match, then we have the correct answer
- That's what a solution means: the value for a pronumeral that makes both sides equal

Writing Equations (page 42)

- Let [the unknown value] be x
- Express the situation using x in an equation
- Solve the equation
- Write the answer using the original "unknown"

- e.g. Toby rented a car for a total cost of \$290. The rental company charged \$40 per day, plus a hiring fee of \$50.

Now: Go back and do the questions from your booklet that you didn't finish